Dynamic Hedging, Expected Returns and Factor Timing

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Equilibrium Asset Pricing: U.S. Equities

- U.S. Stocks
  - Fama-French style factor models explain cross-sectional variation in mean returns
  - Slow moving dividend yields and consumption yields (cay) predict time-series returns
- Legacy modeling frictions
- Common sources of time-series predictability should be linked to common sources of risk

Risk Price Dynamics Should Predict Returns

Target:

Time-varying cross-sectional risk premiums predict time-series returns

RoadMap

Related Work

Alvarez and Jermann (2005), Backus and Chernov (2012), Hansen and Jagganathan (1991), Hansen and Scheinkman (2009)

Santa Clara et. al (2015), Brennan and Taylor (2017), Goyal and Welch (2008), Cochrane (2008), Kelly and Priutt (2013)

Cochrane and Piazzesi (2005), Koijen, Lustig, Van Nieuwerburgh (2017), Fama and French (1993, 2015), Moskowitz and Grinblatt (1999)

Hansen and Richards (1989), Lucas (1979), Harrison and Kreps (1979), Ross (1976), Merton (1972),(1973)

Tool: Markov Dynamics and Decomposition

- Returns $R_{t+1} = r(x_{t+1})$ are functionals of a Markov state x_{t+1}
- Markov transition kernel $(\mathcal{M})_{i,j} := \Pr(x_{[j]}|x_{[i]}) > 0$
- Indexed states are w.l.o.g. $[i]$- indicators $x_{[i]} \mapsto \{0,1\}$
- Rows sum to one $1_{|X|\times 1} =: \mathbf{1} = \mathcal{M}\mathbf{1}$
- If, in addition, we can find $\mu_0 = \mathcal{M}'\mu_0$ such that $\mu_0'\mathbf{1} = 1$,

$$\begin{aligned} \mathcal{M} &= \mathbf{1}\mu_0' + \mathcal{M}_\gamma \\ \mathbb{E}[R_{t+1}|x_t] &= (\mathcal{M}r)'x_t \\ &= (\mathbf{1}\mu_0' r)'x_t + (\mathcal{M}_\gamma r)'x_t \end{aligned}$$

Long Run Mean Index Returns

- Return on total wealth, large time limit

$$\lim_{k\to\infty} \mathbb{E}_t[R_{t+k}] = \lim_{k\to\infty} (r \cdot \mathcal{M}'^k x_t)$$
$$= r \cdot \mu_0 \mathbf{1}' x_t + \lim_{k\to\infty} (r \cdot (\mathcal{M}'_\gamma)^k x_t)$$
$$= r \cdot \mu_0$$

- $r \cdot \mu_0$ measures the long run mean return for bearing aggregate risk (Hansen and Scheinkman (2009), Alvarez and Jermann (2005))
- Washes out predictable variation in expected returns (by construction)

Transitory Component of Returns

- Predictable component of k- period returns driven by $\mathcal{M}_\gamma$,

$$\mathbb{E}_t[R_{t+k}] - r \cdot \mu_0 = r \cdot \left(\mathcal{M}_\gamma'\right)^k x_t$$

- Expected return *factors*

$$\mathbb{E}_t[R_{t+k}] = \underbrace{r \cdot \mu_0}_{\text{permanent}} + \underbrace{r \cdot \left(\mathcal{M}_\gamma'\right)^k x_t}_{\text{transitory}}$$

Persistence is a Feature of Trailing Components

Estimated autoregressions for j'th component y_j, $j = 1, 2$

$$y_{t,j} = \phi_{0,j} + \phi_{1,j} y_{t-1,j} + u_{t,j}$$

coefficient	estimate	$s.e$	$t-$ stat $H_0 : \widehat{\phi}_{i,j} = 0$
$\widehat{\phi}_{0,1}$	0.312	0.624	0.500
$\widehat{\phi}_{1,1}$	0.089	0.072	1.238
$\widehat{\phi}_{0,2}$	-0.059	0.032	-1.836
$\widehat{\phi}_{1,2}$	0.649	0.055	11.841

(a) The first component is not predictable. The second component is significantly positively autoregressive. Fama and French factor return data are quarterly from $Q1$ 1967 to $Q3$ 2015.

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Local Variation in Mean Returns

- Risk premiums vary through time
- Forecasts vary through time (each sufficient for the other)

$$\begin{aligned}
\mathbb{E}_t[R_{t+k}] - \mathbb{E}_{t-1}[R_{t-1+k}] &= \left(\mathcal{M}'\right)^k \Delta x_{t-1} \\
&= \mu_0 \mathbf{1}' \Delta x_{t-1} + \left(\mathcal{M}'_\gamma\right)^k \Delta x_{t-1} \\
&= \left(\mathcal{M}'_\gamma\right)^k \Delta x_{t-1} \\
&=: \Delta \widehat{\mathbb{E}}_{t-1,k}
\end{aligned}$$

- $\Delta \widehat{\mathbb{E}}_{t-1,k}$ always means "changes in conditioning variable for a fixed transition model"

Local Variation in Mean Returns

- Models also vary through time

$$
\begin{aligned}
\mathbb{E}_t[R_{t+k}] - \mathbb{E}_{t-1}[R_{t-1+k}] &= \Delta[\left(\mathcal{M}'\right)^k x_{t-1}] \\
&= \Delta\left(\mathcal{M}'_\gamma\right)^k x_{t-1} + \left(\mathcal{M}'_\gamma\right)^k \Delta x_{t-1} \\
&= \Delta\left(\mathcal{M}'_\gamma\right)^k x_{t-1} + \Delta\widehat{\mathbb{E}}_{t-1,k}
\end{aligned}
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- $\Delta\left(\mathcal{M}'_\gamma\right)^k$ captures "changes in transition model for given conditioning variable"

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RoadMap

Ingredients

- Risk premium dynamics, using $\Delta x_{t+1} = 0$

$$\mathbb{E}_t[R_{t+k}] - \mathbb{E}_{t-1}[R_{t-1+k}] = \Delta \left(\mathcal{M}'_\gamma\right)^k x_{t-1}$$

- Dynamics are o - *distance to Gaussian* (Dyson)

$$\Delta \left(\mathcal{M}'_\gamma\right)^k = -(1-\lambda_2)^{-k}\gamma\gamma' + o\left(\frac{1}{KL(\mathbb{P} \parallel \Phi)}\right)$$

- Time-varying mean returns

$$\mathbb{E}_t[R_{t+k}] - \mathbb{E}_{t-1}[R_{t-1+k}] = -{\zeta_t}^{-k}\gamma\gamma' x_{t-1}$$

- ζ_t is the (log) *spectral gap* of the Markov generator

Identification from Realized Returns

- Principal components of realized returns covariance matrix

$$\mathbb{V}(R) = \mathsf{V}\Lambda_{PCA}\mathsf{V}'$$

- Weak spectral decomposition of Markov return covariance matrix

$$\mathbb{V}(R) = \mathsf{U}D_{1-\lambda}\mathsf{U}'\Sigma$$

$$(D_{1-\lambda})_{i,j} = \begin{cases} (1-\lambda_j)^{-1} & i = j \\ 0 & i \neq j \end{cases}$$

Identification from Realized Returns

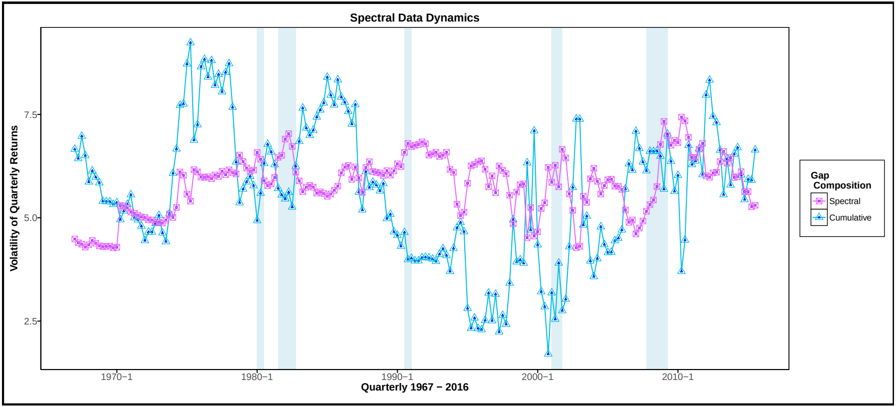
- Cholesky decomposition $CC' = \Sigma$
- Identify the spectral gap up to unitary maps $U^{-1} = U', V^{-1} = V'$

$$\| \mathrm{V}\Lambda_{PCA}^{1/2} \| = \langle \mathrm{U} D_{1-\lambda} C, \mathrm{U} C \rangle$$

- If $I = (CC')^{-1}\Sigma$, with some work we have pointwise identification

$$\zeta^{-1} = (D_{1-\lambda})_{2,2} = (\Lambda_{PCA})_{2,2}$$

Figure: Spectral and Cumulative Gaps: Volatility



(a) The spectral gap measures the difference in terms of volatility of quarterly returns between the leading component and the first trailing component. The cumulative gap measures the difference between the leading component and the sum of all trailing components. Data are quarterly from 1967 $Q1$ to 2016 $Q4$. Fama-French and Carhart factor returns from Ken French. NBER recessions are in blue.

Forecasting Mean Returns

- Now - we use the decomposition and the time-varying spectral gap to forecast returns

RoadMap

Figure: Market Latent Component Dynamics

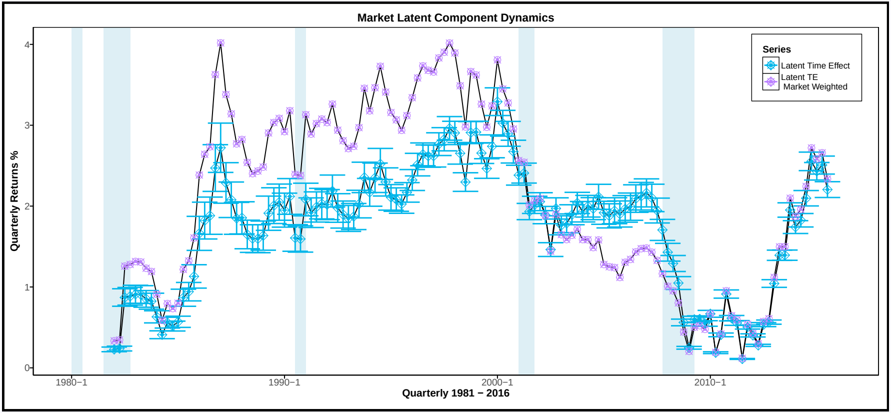


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Table: Market Return Predictability: Spectral Gap and Dividend Yield

Predictor	k (quarters)	coefficient $\widehat{\phi}_{j,k}$	t-statistic	R2	adj.R2
I.	1	2.386	2.770	0.038	0.033
Spectral Gap	2	4.519	3.650	0.065	0.060
	4	8.009	4.805	0.108	0.104
	8	14.755	6.893	0.203	0.199
II.	1	64.066	1.256	0.008	0.003
Dividend Yield	2	143.247	1.929	0.019	0.014
	4	243.685	2.385	0.029	0.024
	8	355.273	2.555	0.034	0.029

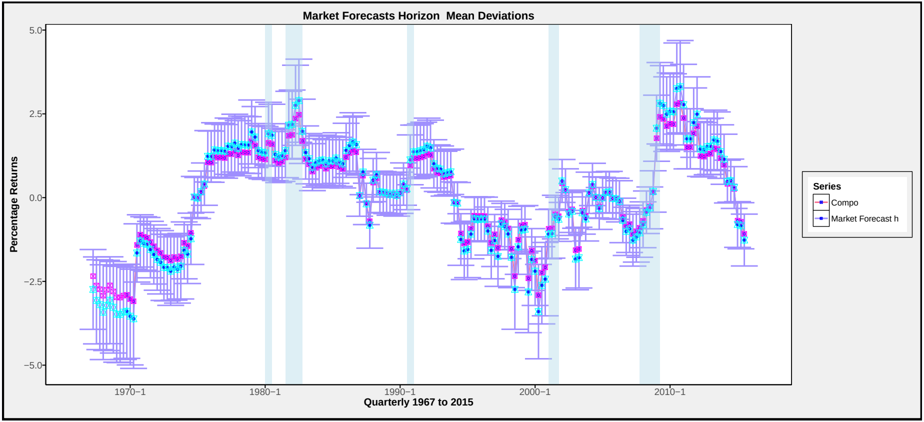
(a) Panel *I* shows out of sample predictability of market returns by the spectral gap. The spectral gap is the difference between the conditional volatilities of the permanent and first transitory factors, measured by the second conditional eigenvalue of the empirical decomposition of asset returns. *II*. shows the out of sample predictability for the dividend yield. The 12-month moving average of monthly dividends, the market index level and *cay* are from Goyal and Welch. Fama and French factor returns quarterly are from Ken French. Data are quarterly from 1967 $Q1$ to 2015 $Q3$.

Table: Market Return Predictability: Spectral Gap and *cay*

Predictor	k (quarters)	coefficient $\widehat{\phi}_{j,k}$	t-statistic	R2	adj.R2
III.	1	47.513	1.944	0.019	0.014
cay	2	101.006	2.821	0.040	0.035
	4	196.512	3.974	0.076	0.072
	8	388.117	5.901	0.157	0.152
IV.	1	1.484	2.136	0.023	0.018
Market Gap	2	2.902	2.895	0.042	0.037
	4	5.297	3.896	0.074	0.069
	8	9.150	5.095	0.122	0.118
V.	1	-35.198	-2.537	0.032	0.027
Asymptotic Gap	2	-63.940	-3.193	0.050	0.045
	4	-118.563	-4.394	0.092	0.087
	8	-228.033	-6.577	0.189	0.184

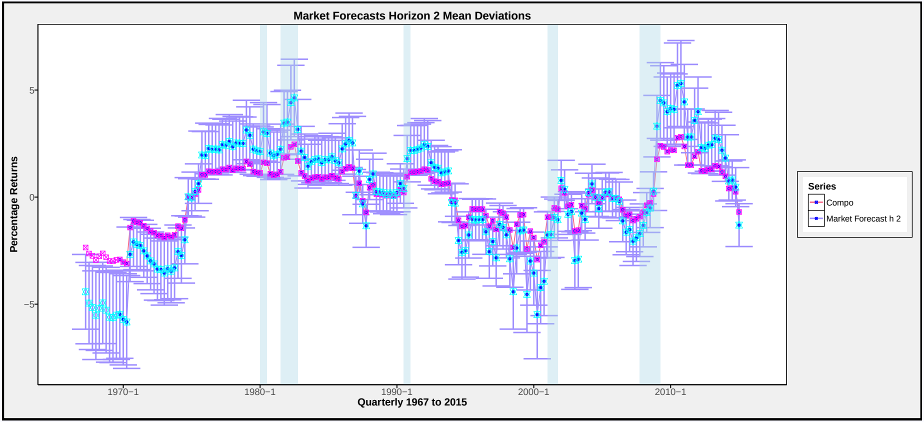
(a) Panel *III*. shows out of sample prediction statistics for *cay*. The spectral gap *excluding* non-market volatility is given in panel *IV*. Panel *V* reports predictability by the transformed gap $s(\Lambda) : \log(s(\Lambda)) = -(1 - \Lambda_{1-2})^{-1}$ written to approach zero asymptotically. *cay* is from Goyal and Welch. Fama and French quarterly factor returns from 1967 to 2015 are from Ken French.

Figure: Time-Varying Expected **Quarterly** Market Returns



(a) Time-series of conditional forecasts, estimated out of sample from the spectral data of a latent Markov model. The spectral data are identified from the principal components of the covariance matrix of realized returns. Quarterly Fama-French and Carhart model returns data 1967 $Q1$ to 2016 $Q4$ from Ken French. Dividends, earnings and *cay* data are from Goyal and Welch 2008. NBER recessions are in blue.

Figure: Time-Varying Expected **Semiannual** Market Returns



(a) The spectral gap is the ratio of the first and second eigenfunctions of the Markov generator. The eigenfunctions correspond to i) the invariant (asymptotic) measure over underlying states, and ii) the empirical transitions, respectively. Quarterly Fama-French and Carhart model returns data 1967 $Q1$ to 2016 $Q4$ from Ken French. Dividends, earnings and *cay* data are from Goyal and Welch 2008. NBER recessions are in blue.

Past Forecast Errors Extend the State Space

- Forecasts cannot be replicated in space of contemporaneous test assets
- Run alternatives separately

$$R_{t+1,j} = a_0 + a_1 \Pi_t + \nu_{t,j}$$
$$R_{t+k,j} = a_{0,k} + a_{1,k} \Pi_{t,j} + u_{t,j}$$

Reject $H_0 : a_{1,k} - a_1^k = 0$ individually, F-test jointly

- Conditional forecast errors are not deterministic functions of horizon k
- Contemporaneous returns are not a Markov state

Table: Value Predictability: Spectral Gap

k (quarters)	variable	estimate	t- statistic	r.squared	adj.r.squared
1	$\sigma(\widehat{\zeta_1})$	-79.5	-1.427	0.010	0.005
2	$\sigma(\widehat{\zeta_1})$	-370.5	-4.598	0.100	0.095
4	$\sigma(\widehat{\zeta_1})$	-576.4	-4.993	0.117	0.112
8	$\sigma(\widehat{\zeta_1})$	-666.7	-4.131	0.085	0.080
1	$\sigma(\widehat{\zeta_2})$	28.84	1.501	0.012	0.006
2	$\sigma(\widehat{\zeta_2})$	125.66	4.508	0.097	0.092
4	$\sigma(\widehat{\zeta_2})$	191.04	4.768	0.108	0.103
8	$\sigma(\widehat{\zeta_2})$	214.77	3.831	0.074	0.069

(a) $\sigma(\widehat{\zeta_1})$ is the volatility of the spectral gap in percentages and $\sigma(\widehat{\zeta_2})$ is the level of the volatility of the spectral gap. Variables are predictors in a linear regression of HML returns. Data are from $Q1$ 1967 to $Q3$ 2015.

Table: Momentum Predictability: Spectral Gap and Earnings

k (quarters)	variable	estimate	t-statistic	r.squared	adj.r.squared
1	$\widehat{\zeta}_1$	-0.460	-1.588	0.013	0.008
2	$\widehat{\zeta}_1$	-0.811	-1.990	0.020	0.015
4	$\widehat{\zeta}_1$	-1.757	-3.090	0.048	0.043
8	$\widehat{\zeta}_1$	-3.463	-4.731	0.108	0.104
1	EP	25.98	1.340	0.009	0.004
2	EP	52.92	1.952	0.019	0.014
4	EP	86.58	2.269	0.026	0.021
8	EP	130.60	2.637	0.036	0.031

(a) Roughly half the fraction of variation is forecastable in momentum returns in comparison to market and HML returns. The gap variable dominates earnings to price ratios, and other common predictors (not reported). Data are from $Q1$ 1967 to $Q3$ 2015.

Replicating Portfolios

- What can returns to the portfolios that replicate our forecasts tell us about **efficiency**?

Table: Sharpe Ratio Comparisons
Value, Momentum, and Size Timing Portfolios

Strategy	Value	Penultimate	Trailing	Value Timing
Monthly SR				
Full Sample	0.177	0.230	0.283	0.301
Pre-2007	0.190	0.259	0.223	0.243
		(3.403)	(2.920)	(3.191)

Strategy	Momentum			Momentum Timing
Monthly SR				
Full Sample	0.2801	0.3283	0.3009	0.3284
				(4.055)

Strategy	Size			Size Timing
Monthly SR				
Full Sample	0.1352			0.2138
				(2.941)

(a) Top: Full sample and pre- 2007 financial crisis Sharpe ratios and t- statistics (pre-crisis) for HML and HML-timing portfolio by component. Mid: Sharpe ratios for momentum and momentum timing, and the first two components of the expected return factors weighted by to their contribution to momentum. Lower panel: size, size timing portfolio and market Sharpe ratios. Test assets are Fama-French FF25 Size/BTM plus 10 momentum portfolios. Factor data are the FF3 plus Momentum, quarterly 1927 $Q1$ to 2015 $Q3$.

Replicating Portfolios

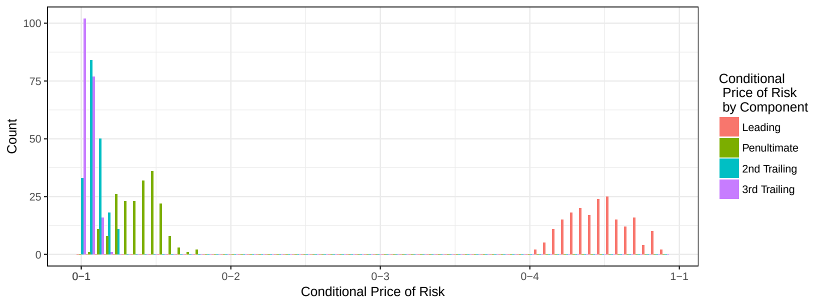
- What can returns to the portfolios that replicate our forecasts tell us about **risk**?

Table: Cross-sectional Pricing
Market and Value Timing Portfolios

coefficient	$\widehat{\lambda}_M$	s.e.	t-stat	$\widehat{\lambda}_V$	s.e.	t-stat	Test Economy
α	0.446	1.592	0.280				FF3 +
Market timing	9.711	2.514	3.862				Market Timing
HML	1.307	0.426	3.066				
Mkt_res	3.618	1.544	2.344				
SMB	0.814	0.506	1.609				
α	0.978	1.738	0.563				Carhart +
Market timing	7.989	2.627	3.042				Market Timing
HML	1.394	0.405	3.444				
Mkt_res	2.721	1.790	1.520				
Momentum	0.662	1.862	0.355				
SMB	0.771	0.504	1.529				
α				2.293	1.702	1.348	FF3+
Value timing				9.982	2.507	3.982	Value Timing
HML_res				-0.486	0.182	-2.671	
MktRF				-0.007	1.593	-0.004	
SMB				0.860	0.508	1.692	

(a) Coefficients are prices of risk. Factors include the market residual Mktres, HML and SMB (top panel) and the market residual, HML, SMB, and Momentum (lower panel). Test assets are the Fama-French FF25 Size/BTM portfolios, with momentum portfolios in the lower panel. Robust standard errors are GMM. Both standard errors and $t-$ stats are reported for convenience. Factor data are the FF3 factor returns. Data are quarterly from 1927 $Q1$ to 2015 $Q3$.

Figure: Empirical Distribution of Risk Prices



(a) Frequencies of conditional (time-series) risk prices for each orthogonalized source of variation. The leading factor represents permanent shocks to the economy, while the penultimate factor prices exposure to unexpected innovations in time-varying expected returns. Expected returns vary predictably throughout the sample. Conditional risk price estimates are the eigenvalues of the conditional covariance matrix of returns and return forecasts. Quarterly data $Q1$ 1967 to $Q3$ 2015.

Discussion: US Equities

Measuring conditional risk prices in equities is valuable

- Expected returns not directly observable
- Complex interaction between different levels of aggregation

But many other markets are relevant. Bonds. Overlapping markets.

RoadMap

The Moral of the Story

Markov structures contribute to *empirical* asset pricing

- The *spectral gap* measures changes in the distribution of priced risk across factors
- Returns (e.g., pricing kernel) decompositions: permanent versus transitory; conditioning versus transitioning
- Conspire to reconcile time-series and risk models:

Time-varying cross-sectional risk premia predict time-series returns

Takeaway: Stylized Facts

- The spectral gap predicts returns out of sample
  - Market: annual o.o.s. R^2 of 10.8%
  - Value: annual o.o.s. R^2 of 11.7%
- Cross-sectional return volatility concentrates countercyclically on permanent shocks to capitalization
- Timing portfolio cross-sectional pricing implications
  - Size is not a risk factor! (Berk)
  - Value is transitory risk
  - Latent market component is significant (CAPM)
  - Momentum is "like" the Market

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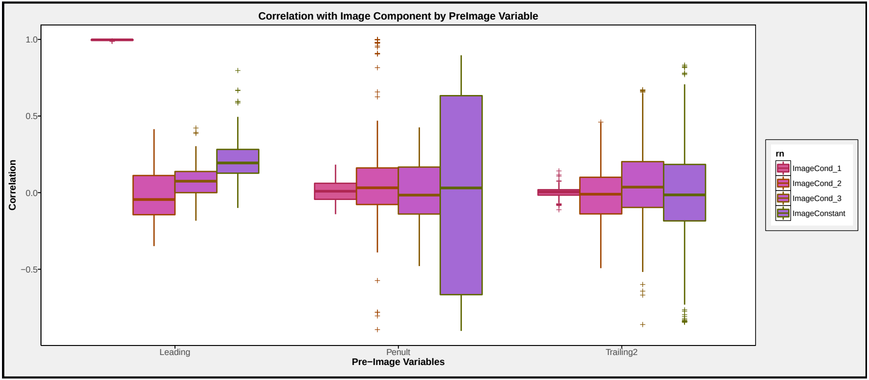
- Cross-sectional return volatility concentrates countercyclically on permanent shocks to capitalization
- *Implications for quantitative models of long-run risk, ambiguity...*

Expected Returns and Factor Timing

Thanks

Note on Mixing Times



(a) Rolling correlations between principal components and smoothed latent factors. The differences between the two are highlighted by the outlying correlations between the PCs and the Markov bases. Downturns in the business cycle are marked by jumps from zero to near one, in absolute value, in the correlation between the trailing components of the two models.